Adaptive Loading in MIMO/OFDM Systems

Prateek Bansal   Andrew Brzezinski

prateek@stanford.edu   brzezin@stanford.edu

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Abstract

Orthogonal Frequency Division Multiplexing (OFDM) is a powerful technique employed in communications systems suffering from frequency selectivity. Combined with multiple antennas at the transmitter and receiver as well as adaptive modulation, OFDM proves to be robust against channel delay spread. Furthermore, it leads to significant data rates with improved bit error performance over links having only a single antenna at both the transmitter and receiver.

This project demonstrates OFDM with adaptive modulation applied to Multiple-Input Multiple-Output (MIMO) systems. We apply an optimization algorithm to obtain a bit and power allocation for each subcarrier assuming instantaneous channel knowledge. The analysis and simulation is considered in two stages. The first stage involves the application of a variable-rate variable-power MQAM technique for a Single-Input Single-Output (SISO) OFDM system. This is compared with the performance of fixed OFDM transmission where a constant rate is applied to each subcarrier. The second stage applies adaptive modulation to a general MIMO system by making use of the Singular Value Decomposition to separate the MIMO channel into parallel subchannels. For a two-input antenna, two-output antenna system, the performance is compared with the performance of a system using selection diversity at the transmitter and maximal ratio combining at the receiver.

I. Introduction

Evolution of OFDM

Frequency Division Multiplexing (FDM) has been a widely-used technique for signal transmission in frequency selective channels. In essence, FDM divides the channel bandwidth into subchannels and transmits multiple relatively low rate signals by carrying each signal on a separate carrier frequency. To facilitate separation of the signals at the receiver, the carrier frequencies are spaced sufficiently far apart so that signal spectra do not overlap. Further, in order to separate the signals with readily sizeable filters, empty spectral regions are placed between the signals. As such, the resulting spectral efficiency of the system is quite low.

In order to solve the bandwidth efficiency problem, orthogonal frequency division multiplexing was proposed, which employs orthogonal tones to modulate the signals [4]. The tones are spaced at frequency intervals equal to the symbol rate and are capable of separation at the receiver. This carrier spacing provides optimum spectral efficiency. Although OFDM was proposed in the 1960's it was not widely employed until the 1990's, largely because of significant circuit design issues, such as spurious frequency components and linearity of amplifiers. Today, OFDM is a major contender for 4G wireless applications with significant potential performance enhancements over existing wireless technology.

Adaptive Loading and MIMO

Adaptive modulation is an important technique that yields increased data rates over non-adaptive uncoded schemes. An inherent assumption in channel adaptation is some form of channel knowledge at both the transmitter and the receiver. Given this knowledge, both the transmitter and receiver can have an agreed-upon modulation scheme for increased performance. In this paper, we consider adaptive bit and power allocation schemes [1], [3]. Namely, we presuppose a desired number of bits to be transmitted by a single OFDM symbol (consisting of N tones), and we load these bits onto the tones in such a way that minimum energy is allocated to the entire transmission.

In addition to adaptive modulation, MIMO is a useful technology with significant data rate improvements of SISO systems. Further to adaptive modulation applied to SISO/OFDM systems, this paper seeks to explore adaptive modulation combined with MIMO/OFDM. A key concept employed here is that every matrix channel can be decomposed into a set of parallel subchannels over which data can be transmitted independently, given appropriate precoding and shaping transformations at the transmitter and receiver, respectively.

Organization of Paper

The paper is structured as follows. Section II introduces OFDM and the key system aspects considered. Section III details the adaptive modulation techniques employed. Section IV explains the new issues introduced by employing MIMO technology and details a generalization of the adaptive technique to MIMO. Section V shows simulation results, and Section VI has conclusions.

II. OFDM System Details

The OFDM system studied in this paper has the block structure as shown in Figure 1. The system assembles the input bits and maps them into complex numbers (in the modulator blocks) which determines the constellation points of each subcarrier. The number of bits assigned to each subcarrier is variable based on the variability of signal to noise ratio across the frequency range. Optimization of this bit assignment will be detailed in further sections. The number of subcarriers N used in an OFDM system is chosen as a trade-off between the frequency offset of adjacent carriers and the adjacent channel interference. A greater number of subcarriers implies less adjacent channel interference, but increased susceptibility to frequency offset, and vice-versa.

FFT and IFFT

The key components of an OFDM system are the inverse FFT at the transmitter and FFT at the receiver. These operations perform reversible linear mappings between N complex data symbols and N complex OFDM symbols. An N-point FFT requires only on the order of $N \log N$ multiplications rather than N^2 as in a straightforward computation. Due to this fact, an OFDM system typically requires fewer computations per unit time than an equivalent system with equalization.

Transmission of data in the frequency domain using an FFT, as a computationally efficient orthogonal linear transformation, results in robustness against ISI in the time domain.
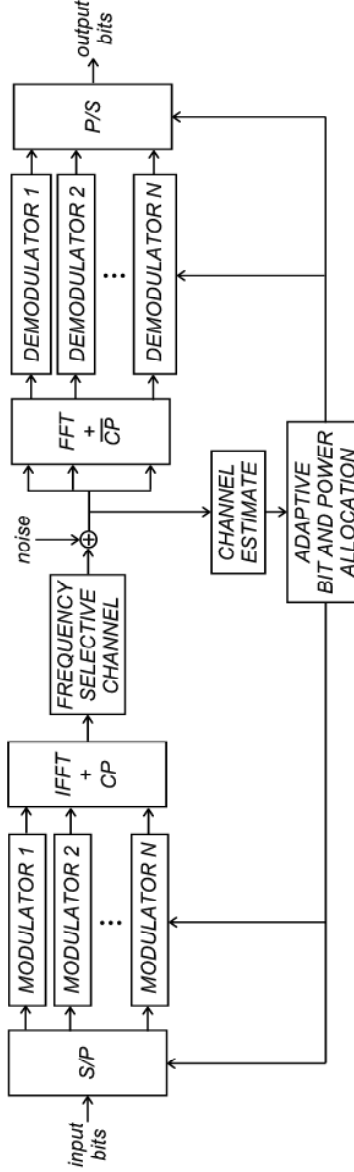
Figure 1: OFDM system block structure

Channel Model and Channel Estimation

Throughout this work, the channel is assumed to be a Rayleigh block fading channel, corresponding to a rich scattering environment with time variation characterized by the fade time. In the MIMO case, the channel is a matrix channel with equation

$$y_n = \sum_{l=0}^{L-1} H_l x_{n-l} + n_n$$

where, in general, the values $\boldsymbol{y}_k, \boldsymbol{x}_k, \boldsymbol{n}_k$ can be vectors, and $\boldsymbol{H}_k$ can be a matrix. Thus, the delay spread of the channel is L symbol periods. An exponentially-decaying profile of channel taps is modeled by fixing the powers of all the elements in each random matrix $\boldsymbol{H}_k$ to a constant E_i. These coefficients E_i form a decaying geometric progression in the variable i. During a coherence time interval, all matrices $\boldsymbol{H}_k$ are constant, and when the channel decorrelates, they are all drawn newly according to their respective pdf's. Further, for simplicity it is assumed that the channel decorrelates at the end of an OFDM symbol transmission.

Channel estimation inverts the effect of non-selective fading on each subcarrier. Usually OFDM systems provide pilot signals for channel estimation. In the case of time-varying channels, the pilot signal should be repeated frequently. The spacing between pilot signals in time and frequency depends on coherence time and bandwidth. Throughout this paper, the channel estimates are assumed to be perfect, and available to both the transmitter and the receiver. Given full knowledge of the channel, the transmitter and receiver can determine the frequency response of the channel, and the channel gains at each tone of the OFDM symbol. Given these gains, the adaptive algorithm can proceed to calculate the optimal bit and power allocation. This step will be expounded in Section III.

Cyclic Prefix

The cyclic prefix is added to an OFDM symbol in order to combat the effect of multipath. Intersymbol interference is avoided between adjacent OFDM symbols by introducing a guard period in which the multipath components of the desired signal are allowed to die out, after which the next OFDM symbol is transmitted. A useful technique to help reduce the complexity of the receiver is to introduce a guard symbol during the guard period. Specifically, this guard symbol is chosen to be a prefix extension to each block. The reason for this is to convert the linear convolution of the signal and channel to a circular convolution and thereby causing the FFT of the circularly convolved signal and channel to simply be the product of their respective FFT's. However, in order for this technique to work, the guard interval should be greater than the channel delay spread. Thus, we see that the relative length of the cyclic prefix depends on the ratio of the channel delay spread to the OFDM symbol duration.

Modulation and Demodulation

A modulator transforms a set of bits into a complex number corresponding to an element of a signal constellation. In this paper, given the adaptive algorithm, the modulator has as input a set of bits and an energy value, so that the output of the modulator is a constellation symbol corresponding to the number of bits on the input, appropriately scaled to have a desired energy.

The modulator is taken to have only a finite number of rates available, which means that only a finite number of constellations are available for the modulation. Specifically these constellations are drawn from the set of constellations having number of symbols equal to an even power of 2. Further, in order to provide robustness against bit errors, Gray-coded constellations are employed for each modulation order available. This Gray coding ensures that if a symbol error occurs, where the decoder selects an adjacent symbol to that which the transmitter intended to be decoded, there is only a single bit error resulting.

Many demodulation techniques can be employed, including maximum-likelihood, MMSE, and zero-forcing. For the paper, in order to simplify the demodulator, demodulation is performed using a zero-forcing approach, given knowledge of the individual flat-fading channel gain for each subchannel.

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III. Adaptive Loading

The advantage of OFDM is that each subchannel is relatively narrowband and is assumed to have flat-fading. However, it is entirely possible that a given subchannel has a low gain, resulting in a large BER. Thus, it is desireable to take advantage of subchannels having relatively good performance; this is the motivation for adaptive modulation. In the context of time-varying channels, there is a decorrelation time associated with each frequency-selective channel instance. Thus, a new adaptation must be implemented each time the channel decorrelates.

The optimal adaptive transmission scheme, which achieves the Shannon capacity for a fixed transmit power is the waterfilling distribution of power over the frequency selective channel. However, while the waterfilling distribution will indeed yield the optimal solution, it is difficult to compute, and it tacitly assumes infinite granularity in the constellation size, which is not practically realizable.

The adaptive loading technique employed in this paper is an efficient technique to achieve power and rate optimization based on knowledge of the subchannel gains ([1],[2]). Only six different square MQAM signal constellations are used; this scheme is expected to perform with efficiency very close to that using unrestricted constellations [8].

In the discrete bit loading algorithm of [1], we are given a set of N increasing convex functions $e_n(b)$ that represent the amount of energy necessary to transmit b bits on subchannel n at the desired probability of error using a given coding scheme. We will assume $e_n(0) = 0$.

The allocation problem which will be using can be formulated as:

Energy Minimization Problem

$$\text{minimize} \sum_{n=1}^{N} e_n(b_n)$$

$$\text{subject to} \sum_{n=1}^{N} b_n = B$$

$$b_n \in \mathbb{Z},\ b_n \geq 0,\ n = 1, 2, \ldots, N.$$

To initialize the bit allocation, the scheme of [2] is employed. The procedure is summarized as follows:

Algorithm Initialization

1. Compute the subchannel signal to noise ratios

2. Compute the number of bits for the ith subchannel based on the formula

$$\hat{b}(i) = \log_2(1 + \text{SNR}(i)/\text{GAP})$$

3. Round the value of $\hat{b}(i)$ down to $b(i)$.

4. Restrict b(i) to take values 0,1,2,4,6 or 8 (correspond to available modulation orders).

5. Compute the energy for the ith subchannel based on the number of bits initially assigned to it using the formula

$$e_i(b(i)) = (2^{b(i)} - 1)/\text{GNR}(i), \text{ where } \text{GNR}(i) = \text{SNR}(i)/\text{GAP}$$

6. Form a table of energy increments for each subchannel. For the ith subchannel,

$$\Delta e_i(b) = e_i(b) - e_i(b - 1) = \frac{2^{b-1}}{\text{GNR}}$$

Consider the kth channel. Given the channel gain and noise PSD, the energy increment table will provide the incremental energies required for the subchannel to transition from supporting 0 bits to 1 bit, from 1 bit to 2 bits, from 2 bits to 3 bits and so on. Since we require our system to have a maximum of 8 bits, the energy increment required to go from 8 bits to 9 bits is set to a very high value. Also, we require the subchannel to have only 0, 1, 2, 4, 6 or 8 bits. Thus, <u>odd number of bits are not supported.</u> In order to take care of this, the energy increment table has to be changed using a clever averaging technique. It is best described by an example:

> Suppose the energy increment required for supporting an additional bit from 2 bits in the nth subchannel is 30 units and that required for supporting an additional bit from 3 bits is 40 units. Then, reassign the energy increment values to the same value, namely, the average of the two. In this case, that value is 35 units. This assures us that if a subchannel is allocated a single bit for going from 2 bits to 3 bits, then in the next iteration the same minimum amount of additional energy required to support another bit will imply that the same subchannel will be allocated the next bit as well.

The same averaging procedure is repeated for all other possible bit transitions. The only exception that might arise is when the algorithm terminates, not having assigned the final bit to even out the total number of bits on that subchannel. In order to resolve this issue, we used an algorithm proposed in [1], (the function ResolveTheLastBit), which will be discussed in the detail later in the section.

Note that we have introduced a new term, GAP. This parameter is in effect a tuning parameter. Different values for GAP yield different E_b/N_o ratios for a given desired number of bits B to transmit. This is because the GAP directly impacts the energy table value calculations. Thus, tuning the GAP allows us to characterize the BER performance of the system.

Given the initial bit allocation, the following algorithm optimizes the bit allocation [1]:

Algorithm B-Tighten

Input:
b, initial bit allocation
B, the total number of bits to be allocated

Output:
b, the optimized bit allocation

Algorithm:
$B' \leftarrow 0$
for $n = 1$ to N

$\quad$ $B' \leftarrow B' + b(n)$

while $(B' \neq B)$

$\quad$ if $(B' > B)$

$\quad\quad$ $n = \arg\max_{1 \leq j \leq N} \Delta e_j(b_j)$
$\quad\quad$ $B \leftarrow B - 1$
$\quad\quad$ $b(n) \leftarrow b(n) - 1$

else
$$m = \arg\min_{1 \le i \le N} \Delta e_i(b_i + 1)$$
$$B \leftarrow B + 1$$
$$b(n) \leftarrow b(n) + 1$$

Finally, in order to deal with a single violated bit constraint, we employ the following algorithm [1]:

Algorithm ResolveTheLastBit

1. Check that the input bit allocation contains at most one violation of the bit constraint

2. If there is a single violation, (say it is in subchannel v), find the bit from the current bit allocation having the largest incremental energy that can be used to fill up subchannel v. Let

$$E1 = \Delta e_v(b(v)) - \Delta e_i(b(i))$$

3. Find the bit that will cost the least to increment in the other subchannels which have been allocated either 0 or 1 bit only. The reason we have this constraint is that all the other subchannels will have 2,4,6 or 8 bits and allocating a single bit to them will violate the bit constraint. Let

$$E2 = \Delta e_j(b(j) + 1) - \Delta e_v(b(v))$$

4. Perform the change corresponding to the smallest of E1 and E2.

Given these three algorithms, we have a complete characterization of the bit loading procedure for a given frequency selective channel.

IV. MIMO/OFDM Systems

MIMO

MIMO systems are defined as point-to-point communication links with multiple antennas at both the transmitter and receiver. The use of multiple antennas at both transmitter and receiver provides enhanced performance over diversity systems where either the transmitter or receiver, but not both, have multiple antennas. This technique can significantly increase the data rates of wireless systems without increasing transmit power or bandwidth. The cost of this increased rate is the added cost of deploying multiple antennas, the space requirement of these extra antennas and the added complexity required for multi-dimensional signal processing.

A great deal of research work has been devoted to the area of combining this spatial scheme with OFDM systems. This system combines the advantages of both techniques in providing simulatneously increased data rate and elimination of the effects of delay spread.

Power control for subchannels on MIMO/OFDM system can be crucial in enhancing the spectral and power efficiency. Without any interference, the best power control to optimize the transmission is the waterfilling solution. But as discussed before, it is not practically feasible and we have employed the above adaptive loading algorithm to characterize the practical performance of MIMO/OFDM system with a single antenna OFDM system.

Analysis of MIMO/OFDM Systems

Consider a MIMO system employing t transmit antennas and r receive antennas. For each tone, the MIMO channel response can be represented by a matrix of size $m \times n$ where the matrix element $h_{j,k}$ represents the channel gain from transmit antenna k to receive antenna j. If we consider the case of perfect channel state information at the transmitter and receiver, we can decompose the MIMO channel on each tone into parallel non-interfering SISO channels using the singular value decomposition (SVD).

Let the instantaneous channel matrix on the ith tone have a singular value decomposition (SVD)

$$\boldsymbol{H}_i = \boldsymbol{U}_i \boldsymbol{S}_i \boldsymbol{V}_i^*$$

where $\boldsymbol{U}_i$ and $\boldsymbol{V}_i$ are unitary matrices, and $\boldsymbol{S}_i$ is the diagonal matrix of singular values of $\boldsymbol{H}_i$. Note that the operator $(\cdot)^*$ is the conjugate transpose operator. Now, if we use a transmit precoding filter of $\boldsymbol{V}_i$ and a receiver shaping filter of $\boldsymbol{U}_i^*$, the equivalent MIMO channel between the IFFT and FFT blocks decomposes into parallel subchannels [8]. Note that the number of such subchannels is exactly equal to the number of nonzero singular values of $\boldsymbol{H}_i$. Denote this number by $c(i)$.

This same decomposition applies to each subchannel of the OFDM system. In general each precoder and shaping matrix will be different for different subchannels.

Adaptive Modulation for MIMO/OFDM

Given the decomposition outlined above, the adaptively modulated MIMO/OFDM system requires that each subchannel have the corresponding precoder and shaping matrix applied to it. Thus, we obtain M effective subchannels, where

$$M = \sum_{i=1}^{N} c(i)$$

In other words, the MIMO/OFDM adaptive modulation problem decomposes into a bit loading over all the nonzero singular values of all the tones. Thus, the problem will be larger than in the SISO case, but the decomposition has allowed us to proceed without any changes to the optimization algorithm.

V. Simulation Results

Assumptions and Simulation Details

Throughout the simulation, the entire system is only considered as a discrete-time system. This simplifies the model somewhat, in that pulse-shaping and matched-filtering are eliminated from consideration. However, these system attributes are relatively simple to incorporate, and do not lead to significant insights beyond those observed with the discrete-time system.

Both a SISO and MIMO simulator were built, and the MIMO simulator was updated to have the SISO system occur as a special case. The following parameters were held constant throughout the simulation:

Number of carriers	64
OFDM symbol time	64 symbol periods
Guard time	16 symbol periods
MQAM available	0, 1, 2, 4, 6, 8
Power delay profile	$[1, 1/e, 1/e^2]$
Noise variance	1×10^{-3}
Coherence time	50 symbol periods

Results

Given the above parameters, simulations were conducted with 100 Monte Carlo iterations for each case.

Bit Allocation

To demonstrate the bit allocation, an instance of the channel was generated and the optimal bit allocation found. Figure 2 shows the channel frequency response, the allocation of bits to each tone, and the corresponding energy on each tone.
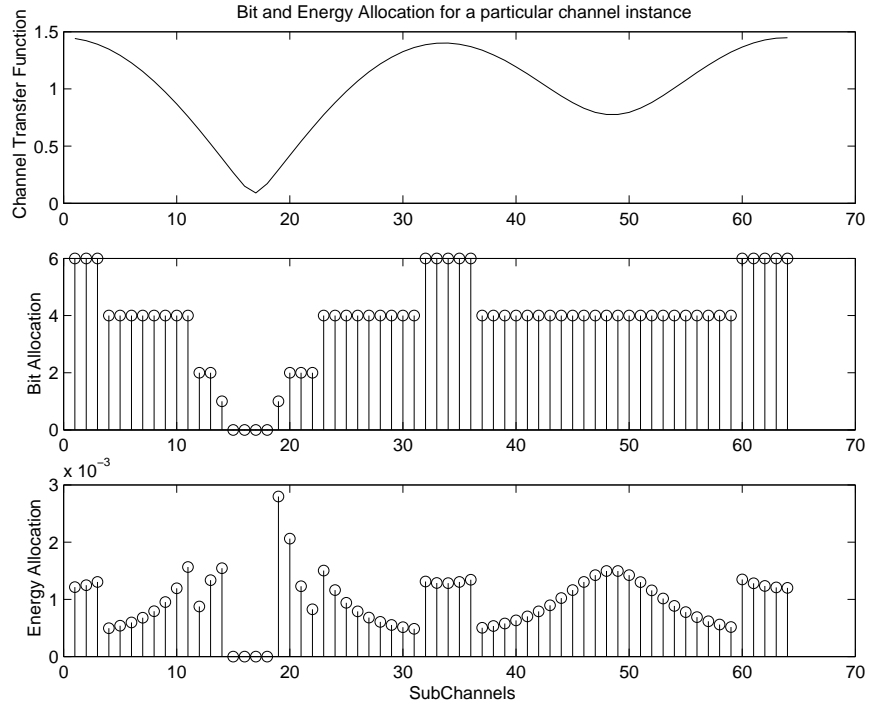

Figure 2: Energy and Bit Allocation for a Channel Instance

As expected, the tones experiencing very poor channel instances had few or zero bits allocated to them. Also, it is interesting to note that the finite number of MQAM constellations available means that the rate remains fixed over some intervals where the gain does not vary too widely.

BER performance

For comparison purposes, the fixed-rate SISO simulator was implemented, where the total number of bits per tone was fixed for all tones, and variable power optimization was applied. The BER performance of the adaptive SISO, adaptive MIMO, and fixed-rate SISO are in Figure 3. In all simulations the MIMO system was held as a 2×2 link. Note that increased averaging (more Monte Carlo iterations) would surely smooth out the BER curves. Clearly, at any given BER the fixed-rate SISO system will be outperformed by the adaptive SISO sytem, which in turn will be outperformed by the adaptive MIMO system. For all three systems, the total number of bits per OFDM symbol were always held constant, to ensure fair comparison.
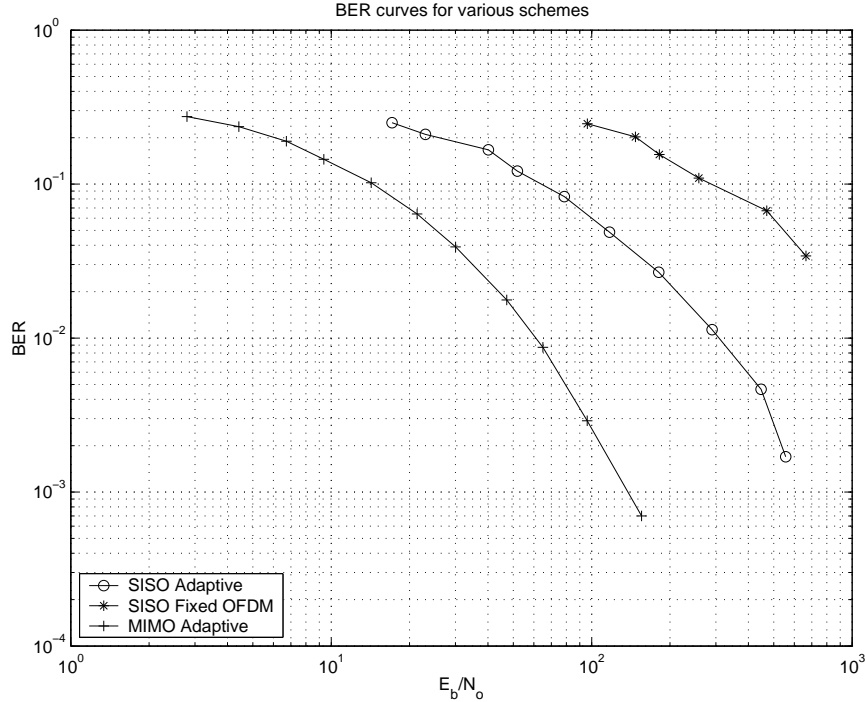

Figure 3: BER curves for various schemes

Finally, a selection diveristy system was considered, where a single antenna at the transmitter was selected based on best achievable rate, and two receive antennas with maximal ratio combining were used for detection of the signal. Figure 4 resulted, with the adaptive MIMO plot provided for comparison.

We see that adaptive MIMO outperforms selection diversity in all cases. This is supported by [3] where the adaptive MIMO/OFDM system performed similarly in all test cases.

VI. Conclusions

We have thoroughly analyzed adaptive optimization algorithms for MIMO/OFDM. We find that the adaptive algorithm employed gives a SISO/OFDM system which outperforms the SISO system having fixed-rate variable-power adaptive modulation. Further, we found that MIMO in general leads to better BER performance, as well as outperforming a common selection combining diversity technique.

We conclude that MIMO/OFDM is a very promising technology, and practical adaptive rate and power optimization algorithms serve well to improve performance. A very useful extension of this paper would be
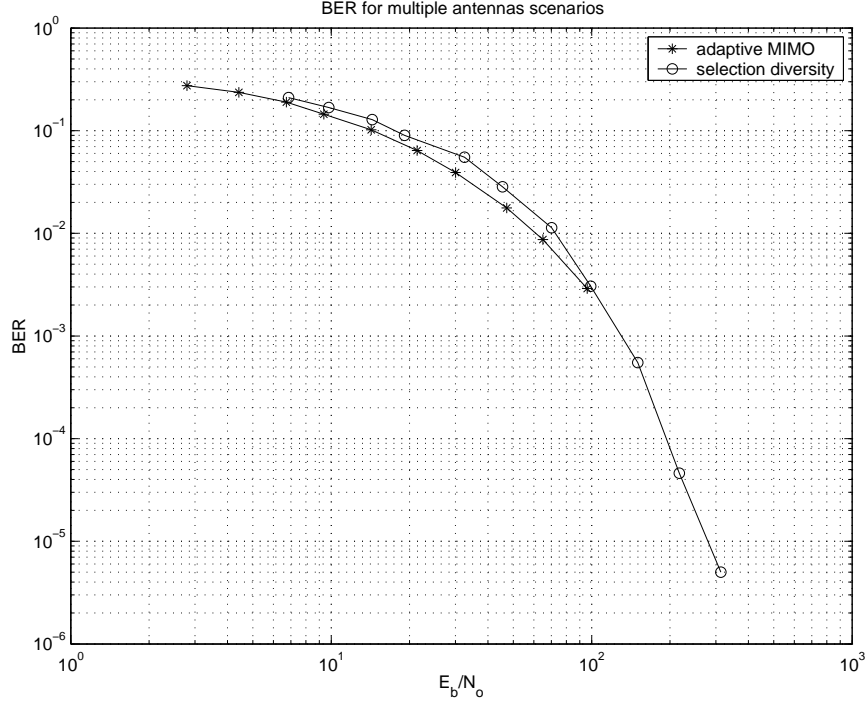
Figure 4: BER curves for various multiple antenna schemes

in multiuser MIMO/OFDM systems, and characterizing good rate and power sharing algorithms to achieve good mutual BER performance of all users, such as in [6] and [7].

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